O Level A Maths Tutorial 11: Applications of Differentiation

Syllabus :
- Derivative as rate of change.

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1. A car travels along a straight road. The distance it covers from time t = 0 is given by

$$s = \tfrac{1}{2} a t^2$$

where $a = 2$ m/s^2.

The rate of change of s is the velocity.

(i) Find the formula for the velocity in terms of a and t. What is the velocity after 10 s.

(ii) The rate of change of velocity is the acceleration. Find the acceleration.

- Use of standard notations f '(x), f ''(x), $\frac{dy}{dx}, \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

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2. (a) $y = x^3 + 2x^2 - 3x + 4$

Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

(b) $f(x) = 2x^4 + 3x^2$

Find $f'(x)$ and $f''(x)$.

- Derivatives of products and quotients of functions.

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3. The product rule is a formula to find the derivative of the product of two functions u(x) and v(x).

$$\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

Use this rule to find the derivative of

$$y=x^2\sqrt{x+1}$$

4. Quotient rule is a formula to find the derivative of a quotient of two functions, u(x) and v(x) :

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

Use this formula to find the derivative of

$$y=\frac{x^2}{\sqrt{x+1}}$$

- Use of chain rule.

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5. The chain rule is used to find the derivative of a function of a function, i.e. a composite function. The function

$$y=\sqrt{x^2+2}$$

can be written as a composite function like this

$$y=\sqrt{u}$$

where we let u be

$$u = x^2 + 2$$

Find the derivative of y with respect to x, using the chain rule :

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$$

- Increasing and decreasing functions
- Stationary points (maximum and minimum turning points and stationary points of inflexion)
- Use of second derivative test to discriminate between maxima and minima

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6. $$f(x) = (x - 1)^2$$

Using the derivative of this function, determine the range of x where f(x) is increasing, and the range of x where it is decreasing.

7. $$f(x) = x^3 - x$$

(i) Find the maximum and minimum turning points. Explain how the second derivative helps to determine these points.

(ii) Find the point of inflextion.

- Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems

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8. Water is poured at a constant rate of 18π cm^3/s into a hemispherical bowl of radius 12 cm. When the depth of water directly below the centre is x cm, the volume in cm^3 of water in the bowl is given by

$$V = 1/3\ \pi\ x^2\ (36 - x)$$

Find

(i) the time taken for the depth of water directly below the centre to reach 9 cm, and

(ii) the rate of change of the depth of water directly below the centre at this time.

[N18/I/7]